Notes Ratio, Proportion and Proportionality

1. Ratio, proportion and proportionality – confusing and often confused
2. Ratios:

    a. A ratio is a comparison of the number or amount of two (or more) things.

    b. Several notations. An example using the quantities of cement and sand in a mortar:

        i. One part cement to three parts sand
        ii. Cement:sand = 1:3 (read as: "cement is to sand as is 1 is to 3")
        iii. $\frac{cement}{sand} = \frac{1}{3}$

    c. Just like rational numbers, a ratio is usually given in lowest terms but can be scaled up to any size by multiplying both numbers by any value. For example, an actual mix of the mortar might be:

        i. Five shovels of cement to fifteen shovels of sand
        ii. Cement:sand = 5:15
        iii. $\frac{cement}{sand} = \frac{5}{15}$

    multiplying both numbers by 5.

    d. Unlike rational numbers, you are not restricted to integers and you can multiply by any number, say 2.5:

        i. Two and a half shovels of cement to seven and a half shovels of sand
        ii. Cement:sand = 2.5 : 7.5
        iii. $\frac{cement}{sand} = \frac{2.5}{7.5}$

    e. Can be more than two things to compare, for example concrete:

        i. One part cement to four parts sand to five parts aggregate
        ii. Cement:sand:aggregate = 1:4:5
        iii. You can't express it in rational number form, but you can pick out ratios so that, in this example, sand:aggregate = 4:5 which can be written $\frac{sand}{aggregate} = \frac{4}{5}$

f. It is important to use the same units when counting or measuring. You can use shovelfuls, bags of the same weight, bags of the same volume, cupfuls, teaspoons – whatever you like. As long as everything is measured the same way, you can use ratios (which have no units) to compare things.

g. Since you can use any numbers in ratios. It is quite common to express ratios such as 4:5 as 1:1.25 (by dividing both numbers by 4).

3. Proportion:

   a. When you have a mixture of things, a proportion is a comparison of the number or amount of one item **to the whole mixture**.

   b. Going back to the mortar example, which was one part cement to three parts sand, there are four parts altogether. So the mix is $\frac{1}{4}$ cement and $\frac{3}{4}$ sand. Similarly, the concrete mix has ten parts altogether and is $\frac{1}{10}$ cement, $\frac{4}{10}$ sand and $\frac{5}{10}$ aggregate. Note that the proportions of sand and aggregate might well be written in their lowest terms: $\frac{2}{5}$ and $\frac{1}{2}$.

   c. Proportions are often expressed in percentages, so the mortar would be 25% cement and 75% sand, and the concrete would be 10% cement, 40% sand and 50% aggregate. Note that the percentages must add up to 100%.

4. Proportionality.

   a. The word ‘proportional’ causes much confusion, because it is not about ‘proportions’ as discussed above, but about the relationship between two things that are logically connected in some way.

   b. For instance, if you are moving at a constant speed, the distance you travel depends upon how long you keep moving. This is written:

   $$Distance \propto Time$$

   which reads ‘distance is proportional to time’

c. There must be some value that determines just how the distance relates to the time, and if we replace

$$Distance \propto Time$$

with

$Distance = \text{‘}some\ number\text{’}\ x\ time$ (in algebra: $d = kt$)

Then, dividing both sides of the equation by the time, we get:

$\frac{Distance}{Time} =' some\ number'$ (in algebra: $\frac{d}{t} = k$)

This ‘some number’ (or k) is called a proportionality constant. In this instance, it is the speed.

d. Note that, unlike a ratio that has no measurement units, a proportionality constant **does** have measurement units. In this case, if the distance were in metres and the time in seconds, the speed would be in metres per second. If the distance were in miles and the time in hours, then the speed would be in miles per hour.

e. What is described so far is direct proportion, in which both things increase together. It is also possible for one thing to decrease as the other increases. This is called inverse proportion. Here is an example. Suppose that you have a strict budget and can only spend £50 per month on fuel for your car. Then the more that fuel costs, the less you can buy This can be written:

$$Litres\ of\ fuel \propto \frac{1}{Price\ per\ litre}$$

In this instance,

$Litres\ of\ fuel = \frac{50}{Price\ in\ \text{£}\ per\ litre}$ (in algebra, $f = \frac{50}{p}$)