

Numbers – Scheme of work

1. Integers:
 - a. Definition:
 - i. Property of sets: counting. Ordered set of names put into 1:1 correspondence. Infinite.
 - ii. Representation on number line
 - iii. Discrete nature: used to count e.g. people, planets, birthdays, visits to places, anything Yes/No
 - b. Four operations and their relationships: inverses, repetition
 - c. Identity operator:
 - i. Importance of zero as identity for add and subtract and as a place-holder
 - ii. Importance of 1 as identity for multiplication and division
 - d. Negative integers. Closure for subtraction. Importance of -1 as the 'change sign' operator in directed numbers.
 - e. Addition and multiplication associative and commutative, subtraction and division are not – use of brackets to disambiguate
 - f. Order of operations and use of brackets to over-ride: BODMAS
 - g. Distribution of multiplication and division across bracketed addition and subtraction and its use in long multiplication
 - h. Division:
 - i. 'Short' and 'long' division algorithms
 - ii. Use of division by factors to avoid long divisions
 - i. Prime numbers:
 - i. Definition and significance.
 - ii. Decomposition of numbers into prime factors
 - iii. HCFs
2. Fractions:
 - a. Closure for division. Representation by pair of integers: numerator and denominator
 - b. Representation on a number line
 - c. Continuous nature: used to measure – needed for science and engineering
 - d. Equivalence sets – lowest terms
 - e. Ordering fractions by expressing with a common denominator
 - f. Fraction operations on proper fractions
 - i. Addition and subtraction by conversion to a common denominator
 - ii. Multiplication: show why denominators are multiplied. Cancelling common factors.
 - iii. Reciprocal: multiplicative inverse
 - iv. Division: multiplication by reciprocal
 - g. Improper fractions and mixed numbers
 - i. Representation on number line
 - ii. Integers as improper fractions

- iii. Conversions
 - iv. Operations with mixed numbers:
 - 1. Addition and subtraction: leave as mixed numbers.
 - a. Dealing with improper fraction result
 - b. Dealing with a negative fraction result
 - 2. Multiplication: convert to improper fractions (show alternative)
 - h. Use of fraction notation to represent division. The fraction line acts like a pair of brackets.
 - i. The definition of the set of rational numbers to include integers. The Pythagoreans. The religious significance of rational numbers – music – the music of the spheres.
3. Decimals
- a. Representation – function of the decimal point and trailing and leading zeros
 - b. Conversion of fractions to decimals
 - i. By inspection and changing to an equivalent fraction
 - ii. By dividing numerator by denominator
 - iii. Recurring decimals
 - c. Conversion of decimals to fractions
 - i. Terminating decimals
 - ii. Warn about error converting rounded recurring decimals
 - iii. Brief mention of algebraic method of converting recurring decimals to fractions
 - d. Decimal operations:
 - i. Addition and subtraction
 - ii. Multiplication: work out the numbers and then adjust the decimal point
 - iii. Division: adjust the decimal point and then work out the numbers
 - e. Accuracy: (note: this does not arise with fractions)
 - i. Integer or power of ten
 - ii. Decimal places
 - iii. Significant figures
 - iv. Implied and explicit accuracy
 - f. Estimation
 - g. Percentages
 - i. Definition
 - ii. Conversion from decimals/fractions
 - h. The definition of the real numbers to include all rationals as terminating or infinitely recurring strings of decimal digits. The existence of irrationals: non-recurring infinite strings of decimal digits.
4. Indices (positive integers only)
- a. Definition and evaluation
 - b. Four operations with indices
 - i. What **not** to do: $a^m + b^m \neq (a + b)^m$ and $a^m + b^n \neq (a + b)^{m+n}$
 - ii. Multiply and divide by adding and subtracting indices iff base the same
 - c. Roots

5. More advanced indices

- a. $x^0 = 1$ **not** $x^0 = 0$
- b. Negative indices – means a root **not** a negative number
- c. Fractional indices: numerator raises to a power, denominator extracts a root
- d. Logarithms i.e. real number indices. Mention four-figure tables and slide rules.
- e. Standard form

6. Ratio

- a. Notation:
 - i. Ratios of two numbers: equivalence of $\frac{3}{4}$ and 3:4
 - ii. Ratios of more than two numbers: e.g. 1: 2: 5
 - iii. Equivalent ratios (note the similarity with equivalence sets of fractions)
 - iv. Extension to use of fractions and decimals in ratios: e.g. $2\frac{1}{2} : 1$ or 2.5 : 1
- b. Use for comparisons
- c. Use for proportion: increase and decrease – direct and inverse
- d. Use of percentages to express ratio
- e. Use for sharing