

Notes on Division of Fractions

Consider the integer division $3 \div 3$. The answer is 1. Let's write this in rational numbers. It must also be true that:

$$\frac{3}{1} \div \frac{3}{1} = \frac{1}{1} = 1$$

But here is a multiplication of $\frac{3}{1}$ by another rational number that also gives an answer of 1:

$$\frac{3}{1} \times \frac{1}{3} = \frac{3}{3} = \frac{1}{1} = 1 \quad \text{or, better} \quad \frac{\cancel{3}1}{1} \times \frac{1}{\cancel{1}3} = \frac{1}{1} = 1$$

So, it seems that $\times \frac{1}{3}$ has the same effect as $\div \frac{3}{1}$. In other words, multiplying by a third is the same as dividing by three. That isn't really a surprise, because taking a third of something means divide it into three equal parts.

What about $\div \frac{1}{3}$ and $\times \frac{3}{1}$? Let's try them out on some number, for example 2 or $\frac{2}{1}$.

We know how to do $\times \frac{3}{1}$:

$$\frac{2}{1} \times \frac{3}{1} = \frac{6}{1} \text{ So the answer is 6.}$$

Is this what we would expect for $\frac{2}{1} \div \frac{1}{3}$? Well, there are 3 thirds in 1, so there would be 6 thirds in 2. So it does work.

What about if we divide a fraction by a fraction? If we divide $\frac{1}{2}$ by $\frac{1}{4}$ we would expect the answer 2, because 2 quarters make a half. So let's try $\frac{1}{2} \times \frac{4}{1}$:

$$\frac{1}{\cancel{2}1} \times \frac{\cancel{2}4}{1} = \frac{2}{1} \quad \text{It works.}$$

So, we conclude that, **to divide by a fraction, multiply by its reciprocal.**

(A reciprocal is the multiplicative inverse which you get by 'turning the fraction upside down' so that, since $\frac{a}{b} \times \frac{b}{a} = \frac{1}{1} = 1$, then $\frac{b}{a}$ is the reciprocal of $\frac{a}{b}$.)